Loudspeakers Mutual Coupling

Keith R Holland
ISVR, University of Southampton, UK
Philip R Newell *Moana, Spain*
http://www.zainea.com/mutualcoupling.htm

The use of loudspeakers in pairs for the reproduction of two-channel stereo gives rise to mutual coupling effects which compound the usual loudspeaker / room interface problems. In this paper, the performance of pairs of idealised loudspeakers in various idealised acoustic environments is discussed and conclusions are drawn concerning the reproduction of centrally-panned phantom images.

1 Introduction

The search for the "perfect loudspeaker" has occupied the minds of audio manufacturers and enthusiasts alike for many years. Most would agree on *many* of the specifications of such a device: a flat frequency response, zero distortion etc., but the requirements for perfect stereo reproduction are far less obvious. Even if such devices existed, the use of these perfect loudspeakers in stereo pairs in real rooms would almost always fail to live up to expectations due to mutual coupling effects which compound the usual loudspeaker / room interface problems.

The influence of room boundary walls on the power output of a loudspeaker has been well researched and documented. In [1], Allison shows how the presence of a single boundary wall increases the power output of a loudspeaker by 3dB at low frequencies, and that introducing two more boundaries gives a net increase of 9dB. More recently, Ward and Angus [2] have extended the concept further to include all six boundary walls. The significance of these findings in the context of this paper is that the presence of a single boundary gives rise to the same sound field as would the introduction of a second, identical loudspeaker placed at the mirror-image position in the absence of the wall. It is therefore logical to assume that introducing a second, identical real loudspeaker - the second of a stereo pair - would also increase the power output of the first loudspeaker. However, whereas the influence of room boundaries on loudspeaker power output is signal independent and may be predicted and corrected for by loudspeaker design and / or electrical equalisation, the influence of one loudspeaker on the other in a stereo pair is *very* dependent upon the exact nature of the (independent) signals fed to the two loudspeakers. For example, when a stereo pair of loudspeakers is reproducing a fully left- or fully tight-panned signal, only one loudspeaker is operating, so, 'perfect' sound reproduction is possible. For centrally-panned images however, both loudspeakers are receiving the same signal, and interference effects give rise to a sound field that is very dependent on position and frequency.

When it is considered that in most modern stereo recordings, much of the important information carrying sounds, such as lead vocals and instruments, narration or dialogue, are panned centrally between the loudspeakers, it is a shame that the reproduction of these sounds is almost always compromised compared to those sounds which are panned fully left or right. It is the objective of this paper to investigate the problems associated with the reproduction of the central phantom image over a stereo pair of loudspeakers, and to attempt to shed some light on how these problems may be reduced or overcome.

To attempt to analyse the entire loudspeaker pair / room interface problem would be an enormous task and would yield results that only really applied to the particular set of conditions being modelled; what is required is a simpler, more general approach with the number of variables reduced to a minimum. To this end, this paper is concerned with examining the performance of idealised "perfect" loudspeakers when used in pairs for reproducing stereo signals, with particular reference to the reproduction of the central phantom image. The behaviour of velocity-source and pressure-source loudspeaker pairs is analysed in some detail under both freefield (anechoic) and reverberant acoustic conditions.

2 The Analysis of Perfect Loudspeakers in Perfect Rooms

Consider two idealised acoustic environments - the anechoic chamber and the reverberant chamber - both containing a single, perfect, omnidirectional, velocity-source loudspeaker (loudspeaker A). The response at any point in the anechoic chamber is dependent upon the pressure response of the loudspeaker (the sound pressure at a point per unit electrical input) which, as the loudspeaker is omnidirectional, is the same everywhere. In the reverberant chamber, the response at any point is the sum of an infinite number of reflexions from the walls, which all arrive with different time delays (a diffuse field). The reverberant response therefore depends upon the power response of the loudspeaker (the total sound power radiated per unit electrical input), and is thus also the same everywhere. The power output of any source can be found by integrating the anechoic responses over all angles, so the sound power response of an omnidirectional loudspeaker is the same as the pressure response; a flat response will result everywhere in both the anechoic chamber and the reverberant chamber.

2.1 Two Velocity-Source Loudspeakers in an Anechoic Chamber

Now introduce a second perfect loudspeaker into the anechoic chamber (loudspeaker B) and feed it with the same signal as the first, such as is the case for a centrally panned stereo image. The response will no longer be flat everywhere because of the interference between the sound fields radiated by the two loudspeakers - the combined output is no longer omnidirectional. However, at any point along a centre-line, equidistant from the two loudspeakers (the stereo 'hot-seat' line), the two sound fields will add constructively at all frequencies giving a flat response 6dB higher in level than that of one loudspeaker alone. At all other points, the sound fields will constructively or destructively interfere depending on the path length differences and the wavelength (frequency) of the sound - a comb-filtered response will result. The sound field can be calculated as the sum of the pressures generated by the two loudspeakers and this pressure can be compared to that generated by a single loudspeaker of the same output, placed midway between the pair:

$$\frac{p_2}{p_1} = \frac{R}{r_A} e^{-jk(R-r_A)} + \frac{R}{r_B} e^{-jk(R-r_B)} , \qquad (1)$$

where R, rA and rB are the distances from the point of interest to the central loudspeaker, loudspeaker *A* and loudspeaker *B* respectively (see figure 1) and $k = \omega / c_0$ is the free-space wave number at an angular frequency of m radians per second. Figure 2 shows two typical responses of a pair of loudspeakers at positions away from the centre-line relative to that of a single loudspeaker placed midway between the pair.

2.2 Two Velocity-Source Loudspeakers in a Reverberant Chamber

If we introduce the second loudspeaker into the reverberant chamber. The response is now the same everywhere and is dependent upon the *combined* power response of the two loudspeakers. A simple realisation of an omnidirectional source is that of a pulsating sphere. The sound power radiated by such a source can be written

$$W = \frac{S}{2}\,\Re\{p(a)\,u(a)^*\} \;, \qquad (2)$$

where a is the radius of the sphere, u(a) is the surface velocity of the sphere, p(a) is the acoustic pressure on that surface, S is the surface area, 9t{) denotes the "real part of and' denotes the complex conjugate. For a pair of velocity-source loudspeakers, u(a) is fixed and p(a) is the sum of the pressure generated by loudspeaker A due to its own velocity and that generated by loudspeaker *B* on the surface of A. It should be noted that only the direct sound from *B* affects the power output in the reverberant chamber - the reverberant field is assumed to be diffuse and therefore has random phase and a net effect of zero; the power output of the loudspeaker pair is therefore the same as under anechoic conditions. The combined power output of the pair of loudspeakers, relative to that of a single loudspeaker is then

$$\frac{W_2}{W_1} = 2\left(1 + \frac{a}{d}\cos\big(k(d-a)\big) + \frac{1}{kd}\sin\big(k(d-a)\big)\right) \;\approx 2\left(1 + \frac{\sin(kd)}{kd}\right) \text{ if } (d \gg a) \;, \qquad (3)$$

where d is the distance between the two sources. A derivation of equation (3) can be found in the appendix. Figure 3 shows the combined power output of a pair of velocity-source loudspeakers relative to the power output of one of the loudspeakers operating in isolation.

The important features to note about figure 3 are that, in agreement with [1] for a single loudspeaker and reflective wall, at high frequencies the power output of the pair of loudpeakers is approximately +3dB (double) relative to that of a single loudspeaker - entirely as expected, and that at low frequencies the increase in power output is nearer +6dB (four times). The "magic" doubling of power output at low frequencies may be explained using the concept of mutual coupling.

2.3 Mutual Coupling

The concept of mutual coupling between loudspeakers is familiar to anyone who has mounted two loudspeakers close together. The power output of the two loudspeakers is approximately four times (+6dB) that of a single loudspeaker. Also, if you double the area of the diaphragm of

a loudspeaker drive-unit, given the same diaphragm velocity, the power output will again increase by +6dB. Reference to equation (2) shows that introducing a second loudspeaker close to a first will approximately double the pressure on each of the diaphragms, thereby doubling the power output of both loudspeakers.

What is perhaps less obvious however, is how introducing *a distant* second loudspeaker can double the power output of a loudspeaker. For the 3m separation and 0.15m radius of the pair of loudspeakers in the above examples, the magnitude of the pressure on loudspeaker A due to the operation of loudspeaker B is approximately one twentieth of the pressure on A due to its own velocity. How can an increase in pressure of 5% cause a doubling of power output? The answer lies in the phase of the two pressures. At low frequencies, the pressure on the surface of A due to its own velocity is almost in phase quadrature with the velocity - the radiation impedance is almost totally reactive - whereas that from *B* arrives almost *in-phase* with the velocity due to the propagation distance involved. Equation (2) tells us that it is only the in-phase part of the pressure that is responsible for power output. As the distance d is decreased, the magnitude of the pressure due to the second source increases but its phase approaches that of the pressure due to the velocity of the first source - the power increase remaining at +6dB but extending higher in frequency - until the "two close loudspeakers" situation exists. As can be seen from equation (3), the frequency up to which the mutual coupling occurs is determined by the distance between the two sources; as the propagation distance approaches half a wavelength the phase of the pressure from the second source is no longer in phase with the velocity. The distance over which mutual coupling occurs is known as the extent of the hydrodynamic near field of the loudspeakers.

2.4 Transient Signals

The concept of mutual coupling is fine for explaining the power increase with steady-state, single tone signals. However, under transient excitation, the two loudspeakers operate simultaneously and by the time the pressure from loudspeaker *B* has reached loudspeaker A, loudspeaker *A* has stopped moving. If they are velocity-source loudspeakers, the presence of the delayed transient pressure from B can have no effect on the sound radiated from A, as this transient has already left the loudspeaker. However, the steady-state and transient responses of any linear system are linked by the Fourier transform pair, so any change in response to steady-state excitation *must* be reflected in the transient response, so what has happened to the mutual coupling with transient excitation?

In order to explain the transient response it is necessary to study the steady-state directivity of the loudspeaker pair. Figure 2 shows the response of a pair of loudspeakers at two different positions away from the centre-line in an anechoic chamber. Two important features of the two responses are that the peaks and dips in response occur at different frequencies for different positions and that the responses are similar at low frequencies. It can be shown that the combined power response of the two loudspeakers is proportional to the sum of the (squared) anechoic responses over all angles, ie the sum of an infinite number of responses of which those in figure 2 ace typical examples. At low frequencies, all of the responses are similar and they sum to give +6dB increase in power output compared to a single loudspeaker. At higher frequencies however, the net result of summing all of the (different) comb-filtered responses is, on average, a +3dB increase compared to a single loudspeaker. The result of integrating the squared responses (actually the intensities) over all angles is therefore the power response shown in figure 3. Figure 4 shows the (far-field) polar directivity response of a pair of loudspeakers separated by 3m. It can be seen that integration of the (squared) polar diagrams would yield a result close to 2 (+6dB) at low frequencies and approximately 1.4 (+3dB) at high frequencies. Thus the mutual coupling phenomenon can be explained easily in terms of the directivity of the loudspeaker pair.

At positions along the centre-line between the two loudspeakers, the transients from the two loudspeakers arrive together and superimpose perfectly giving a transient of double the height; +6dB at all frequencies. At all other positions, they are time-displaced and therefore do not sum to give a double height transient. The Fourier transform of a double transient signal is a comb filtered response like those shown in figure 2. Integration of the intensity of the double transient over all angles therefore yields the same result as for the steady-state response, but integrated over all of the frequencies contained within the transient signal. It is clear therefore that narrowing the transient in time increases the bandwidth, narrows the angle in space over which the two transients overlap, and reduces the significance of the low frequency gain to the overall power.

2.5 Pressure-Source Loudspeakers

The above discussion on directivity and transient response would seem to indicate that replacing the velocity-source loudspeakers, for which the source velocity is independent of the pressure load exerted upon it, with pressure-source loudspeakers, for which the velocity changes with changing pressure load so as to maintain constant pressure, would have little effect on the combined power output. However, consideration of the mechanism of mutual coupling suggests that the increase in pressure due to a second loudspeaker would cause a reduction in velocity which would reduce power output. The derivation of the equivalent of equation (3) but for pressure-source loudspeakers is also in the appendix and this shows that the combined power output of a pair of pressure source loudspeakers, relative to that of a single loudspeaker is

$$\frac{W_2}{W_1} = 2\frac{\left(1 + \frac{a}{d}\cos(k(d-a)) + \frac{1}{kd}\sin(k(d-a))\right)}{1 + \left(\frac{a}{d}\right)^2 + 2\frac{a}{d}\cos(k(d-a))} \approx 2\left(1 + \frac{\sin(kd)}{kd}\right) \text{ if } (d \gg a) \, , \qquad (4)$$

thus in the limit of *d » a,* pressure-source loudspeakers *do* demonstrate mutual coupling to the same degree as velocity-source loudspeakers. However, as the source size is increased or the spacing between the loudspeakers is decreased, the mutual coupling reduces until it is zero for the "two close loudspeakers" case, and only a +3dB power increase is observed. The small additional pressure load exerted by a distant second loudspeaker only changes the velocity by a small amount, despite the increase in power output, but the doubling of the pressure load exerted by a close second source reduces the velocity to one half. Figure 5 shows the combined power output of a pair of pressure-source loudspeakers of the same size and spacing as the velocity source loudspeakers shown in figure 3, and figure 6 shows the combined frequency response at 20° away from the centre-line to compare with the first plot in figure 2.

3 Discussion and Practical Implications of Results

The above analysis of the mutual coupling between a stereo pair of perfect loudspeakers in ideal environments is interesting from an academic point of view, but how do the results relate to the usual situation of imperfect loudspeakers in imperfect rooms? When a single loudspeaker is operated in a typical room, mutual coupling occurs between the loudspeaker and each of the mirror image loudspeakers in each wall; there is also mutual coupling between each mirror image and each other mirror image and so on... So to worry unduly about the coupling between the two loudspeakers in a stereo pair seems, at first thought, a bit silly. However, the main objective of this paper is to investigate the phantom central image, and it is in the reproduction of this that mutual coupling between the loudspeakers themselves becomes important. If there are a certain number of significant, coupled sources in a given room when one loudspeaker is operated, this number will always at least double when two loudspeakers are operated; most rooms behave in a more or less semi-reverberant manner, so the combined power output of the loudspeakers is of importance. Thus there will always be a significant difference between the reproduction of a fully left- or right-panned image and the centrally-panned phantom image.

3.1 The Pan-Pot Dilemma

In an anechoic chamber, a mono-eared listener sat directly on the centre-line between a stereo loudspeaker pair, listening to a broad-band sound that is panned from fully left, through centre, to fully right, hears no change in timbre or level (head related transfer functions apart) if the panpot reduces the signal level by -6dB to each loudspeaker in the central position. If that listener moves away from the centre-line, the sound will be perceived as going from "flat" through "coloured" to "flat" as it is panned, due to the poor directivity of the stereo pair giving rise to comb-filtering (see figure 2); the nature of the coloration being different for different off-centre positions. This would, of course, be accompanied by the usual "break down" of the stereo illusion associated with off-central listening. The only problem experienced by a listener in the hot-seat in an anechoic chamber is that most of us have a head with a "working" ear on both sides, so neither ear is on the centre-line. Also, the diffraction around the head is different for a single frontal source than for a phantom image. The comb-filtering associated with having ears that are some 100mm away from the centre-line has its first dip at around 2kHz under typical listening conditions.

In the reverberant chamber, the same signal would be perceived as having the same "flat" spectrum as in the anechoic chamber when panned fully left or fully right (any information content in the signal would be severely masked by the reverberation, however), but when the signal is panned to the central position, it has a spectrum similar to that in figure 3, with a +3de rise at low frequencies. If the same -6dB pan-pot law is used, the low frequency content of the signal will remain the same, as it is panned, but the rest of the signal will be reduced in level by -3dB at the central position. One can imagine a suitable pan-pot law for use in the reverberant chamber which reduced the low frequencies by -6dB and the higher frequencies by -3dB at the central position, thus maintaining the "flat" spectrum at all positions of the panned image, but listening to stereo reproduction in a reverberant chamber has limited appeal!

Clearly, the ideal pan-pot law depends upon the acoustics of the room in which the sound will be reproduced. Real rooms behave in a manner somewhere between anechoic and fully reverberant, so some compromise is necessary. Many mixing console manufacturers will produce different pan-pot laws for different applications, though they usually opt for a -4'hdB compromise, which produces only a 1'hdB worst case error. The fact that this seems to work well is borne out by the number of recording engineers who fail to realise that this situation exists at all. Regardless of the pan-pot law chosen, in all situations, with the notable exception of a mono-eared listener sat in the stereo hot-seat in an anechoic chamber, a centrally-panned phantom source will be perceived as having a different timbre from a true centrally mounted loudspeaker; a fact that has strong implications for stereo / mono compatibility (see section 3.4). Amongst experienced recording engineers, there is a saying: "pan first, then equalise"; there is wisdom in this statement, even in perfect acoustic environments.

3.2 Specialist Listening Room Design –

The Studio Control Room

The importance of mutual coupling and the related directivity problems associated with a stereo pair of loudspeakers (see figure 4) depends to a large extent on the acoustic treatment of a listening room. The domestic end users of much recorded material usually have little control over the acoustics of their listening environment, and many people these days listen via headphones or in automobiles where the problems associated with "normal" stereo reproduction do not occur. However, it is in the recording studio where the creation and quality control of the recording is carried out, and it is in the control rooms of these studios where the problems associated with stereo pairs of loudspeakers are important, and where specialised acoustic design is possible.

The anechoic chamber seems to offer the best situation, provided listening is carried out in, or near, the stereo hot-seat. The authors have been lucky to have experienced stereo reproduction over high quality loudspeakers in the large anechoic chamber at ISVR;

it is a memorable experience.

In truly reverberant condition, the total sound power output of the pair of loudspeakers is maintained in the reverberant field, irrespective of the fact that the interference between the loudspeakers causes different, comb-filtered responses in different directions (see figures 2 and 4). However, once absorption is introduced into the room, the absorbent areas rob the reverberant field of energy, either direct or reflected, which travels in the direction of the absorbent surfaces. This absorbed energy will be non-uniform in frequency content - even with "perfect" absorbers - due to the poor directivity of the loudspeaker pair. Likewise, if reflective surfaces are introduced into an otherwise "dead" room, they return energy to the listener, once again, with a frequency balance which is dependent upon the directivity of the loudspeaker pair. These problems do not occur when only one omnidirectional loudspeaker is operated. So, even in a world with perfect reflectors, absorbers and diffusers, we still could not produce "accurate" listening conditions for the central phantom image between two perfect loudspeakers in nonanechoic rooms.

The "sister" of this paper "A Proposal for a More Perceptually Uniform Control Room for Stereophonic Music Recording Studios" by Newell &

Holland [3].[in our site also] puts forward strong arguments for what are termed "Non-Environment" rooms. The idea behind these rooms is that the side walls, rear wall and ceiling are made as absorbent as possible down to as low a frequency as possible whilst the front wall and floor are hard and reflective. The monitor loudspeakers are mounted in the hard front wall, providing a rigid, diffraction-free baffle, and this surface along with the hard floor also provide the "acoustic life" desired by the room occupants. Such a room is ideal for optimised reproduction of the central phantom image. The rooms are not at all reverberant, so the mutual coupling problem does not occur, and the sound leaving the monitors can only be reflected off of the floor which, being a horizontal surface, doe not reflect the comb-filtered response to the listener.

3.3 Real Loudspeakers

So far we have considered only "perfect" spherical omnidirectional loudspeakers. Most real loudspeakers are omnidirectional at low frequencies but are far from omnidirectional at higher frequencies. The low frequency mutual coupling argument can be applied to real loudspeakers with some confidence however. Equation (3) for example, only requires a small change in its derivation for adaptation to baffled pistons in place of pulsating spheres, and is identical in the limit of d » a. Consideration of pressure-source loudspeakers shows that the finite mechanical impedance of real loudspeakers also has little effect on the results. The directivity of real loudspeakers at mid and high frequencies significantly alters the directivity of the loudspeaker pair however, and this changes the interaction with absorbers and reflectors, as discussed in section 3.2. As a general rule, narrow directivity loudspeakers interact less with room acoustics than wide directivity loudspeakers, whether considering a stereo loudspeaker pair or simple mono reproduction. Under anechoic listening conditions, there are no reflexions, so the only sound heard by a listener is that which passes directly from the loudspeaker to the listener, therefore there is no perceivable difference between a perfect omnidirectional loudspeaker and one which radiates a uniform frequency response only in the direction of the listener; the omnidirectional loudspeaker is just wasting power.

Dipole loudspeakers, such as most electrostatics, behave in a different manner. The dipole radiation pattern means that little or no sound is radiated towards the other loudspeaker thus rendering them immune to mutual coupling effects providing the stereo pair do not face each

other. Some room-related mutual coupling will still occur however, although to a lesser extent than for monopole loudspeakers.

3.4 Stereo / Mono Compatibility and Surround Sound

Under almost all stereo listening conditions, mutual coupling gives rise to a change in timbre of a sound as it is panned from fully left or right, to centre. Given that most listening rooms can be described as semi-reverberant, there may be a frequency equalised pan-pot law that could apply some correction to the low frequency boost of the central image (although *not* the directivity problems). The frequency below which this low frequency cut should occur is determined by the distance between the loudspeakers, and the amount of cut is related to the low frequency reverberation time of the room. However, if such a pan-pot were used, the resulting mix would not work correctly under non-average stereo listening conditions such as headphones, portable stereos ("boom boxes") and in-car audio. What is probably more important though, is how such a mix would "fold down" to mono. The correct pan-pot law to use for mono compatible stereo is the -6dB, voltage summing law described in section 3.1. Any stereo mix that attemps to correct for mutual coupling under stereo listening conditions will not be correct when summed to mono. This situation is compounded when multi-channel surround sound systems are considered. Figure 7 shows the combined power output of four loudspeakers arranged in a rectangle of 3m x 4m. What is immediately apparent is that the mutual coupling problem associated with a stereo pair of loudspeakers is compounded with four loudspeakers to give a low-frequency boost of +12dB (16 times more power) compared to a single loudspeaker. Remembering that this boost will depend upon the room acoustics and the type of loudspeaker used, what form should the surround sound mix take, and how will such a mix fold down to stereo or even mono? This is a question of great importance for television.

One possible solution to the multi-channel compatibility problem, and that of mutual coupling in general, could be the use of a mono subwoofer. The subwoofer could reproduce the low frequencies below the half-wavelength frequency thus eliminating the low frequency boost in centrally-panned sounds. One problem with using a mono subwoofer is that the path length from the mid-frequency loudspeakers and that from the subwoofer will be different at different points in the room, giving rise to possible crossover / localisation problems. A solution to this would be to keep the low frequency loudspeakers in their stereo positions and to connect them together electrically. The main disadvantage with the mono subwoofer though, is that it cannot reproduce any out-of-phase low frequency stereo information. Such information, even if not reproduced faithfully (a very rare situation), can contribute to the feeling of ambience in live recordings or enhance special effects in film soundtracks etc.

Most of the problems described in this paper are greatly reduced by the adoption of a third, centre channel. The important, information carrying central image would then be reproduced with the same quality as the fully left or right images. Ideally, a new three-channel version of stereo could be introduced, but existing centre-channel systems, such as those adopted in some surround sound systems, can be effective. An additional bonus to the use of a centre channel is an effective widening of the stereo hot-seat, at least for centrally-panned images.

A completely different approach to stereo reproduction may also alleviate the problems. Such a system, using a pair of closely-spaced loudspeakers, is being researched by Kirkleby et al [4]. The close proximity of the two loudspeakers means that any mutual coupling that does occur, does so over a wide frequency range, and that the directivity of the pair of loudspeakers is greatly improved.

4 Conclusion

This paper is concerned with two problems associated with the reproduction of the important phantom central image via a stereo pair of loudspeakers. One problem concerns the increase in power output of a loudspeaker when a second loudspeaker is fed with the same signal, and the other concerns the poor directivity of a widely-spaced loudspeaker pair and the interaction of this directivity with the listening room.

Studies of the behaviour of pairs idealised loudspeakers under idealised acoustic conditions show that the two problems share the same cause but have different effects.

It is concluded that under almost all listening conditions the timbre of a signal panned centrally between a stereo pair of loudspeakers will differ from that from a centrally mounted mono loudspeaker (or that from either of the stereo loudspeakers alone). The conclusions drawn are shown to have a direct bearing on recording studio control room design as well as compatibility problems between multi-channel, stereo and mono systems, and serve to highlight the fact that two-speaker stereo is a very unstable illusion.

5 References

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[2] T Ward and J A S Angus, "The Effect of a 6 Walled Room on Loudspeaker Output", Proceedings of the Institute of Acoustics, Vol. 18, part 8, Reproduced Sound 12, 1996, p253-61.

[3] P R Newell and K R Holland, "A Proposal for a More Perceptually Uniform Control Room for Stereophonic Music Recording Studios", presented at the 103rd Convention of the Audio Engineering Society, September 1997.

[4] O Kirkleby, P A Nelson and H Hamada, "The "Stereo Dipole" - Binaural Sound Reproduction using Two Closely Spaced Loudspeakers", presented at the 102nd Convention of the Audio Engineering Society, March 1997, AES Preprint No. 4463

Appendix

Derivation of Combined Power Output Equations

Consider a perfect spherical source of radius a pulsating at a frequency ω. For a free, progressive spherical expanding wave field, the pressure at any radius r can be written,

$$p(r) = \frac{Ae^{-jkr}}{r} \ , \tag{A1}$$

where A represents an arbitrary amplitude. According to the momentum equation, the velocity field can be expressed as,

$$j\omega u(r) = \frac{-1}{\rho_0}\frac{\partial p(r)}{\partial r} \ , \tag{A2}$$

thus the pressure on the surface of the source due to its own surface velocity $u(a)$ is,

$$p(a) = \frac{\rho_0 \, c_0 \, u(a)}{(1 - j/ka)} = Z_{r0} \, u(a) \ . \tag{A3}$$

where Z_{r0} represents the free-space radiation impedance. Combining equations (A1) and (A3), the pressure at any distance d from the source can be written in terms of the surface velocity,

$$p(d) = Z_{r0} \, u(a) \, \frac{a}{d} \, e^{-jk(d-a)} \ . \tag{A4}$$

When two identical sources separated by a distance d are operated simultaneously, assuming that the sources are compact compared to a wavelength, the total pressure on the surface of either of the sources is,

$$p(a) + p(d) = Z_{r0} \, u(a)\left(1 + \frac{a}{d} e^{-jk(d-a)}\right) \ . \tag{A5}$$

The sound power output of a source may be written,

$$W = \frac{S}{2} \, \Re\{p(a) \, u(a)^*\} \ , \tag{A6}$$

where S is the surface area of the source, $\Re\{\,\}$ denotes the "real part of" and * denotes the complex conjugate. The ratio between the sound power output of a single source in isolation and that of one of a pair of sources is therefore

$$\frac{W_2}{W_1} = \frac{\Re\left\{\left(p_2(a) + p_2(d)\right) u_2(a)^*\right\}}{\Re\{p_1(a) \, u_1(a)^*\}} \ , \tag{A7}$$

where the suffix $_2$ refers to velocities and pressures with both sources operating and the suffix $_1$ refers to a single source operating in isolation, thus $p_2(a)$ is the pressure on the surface of one source due to its own velocity $u_2(a)$ with both sources operating, $p_2(d)$ is the pressure on one source due to the velocity of the other, etc.

Constant velocity sources

If the sources are assumed to operate as constant velocity sources, the surface velocity is independent of any external pressure exerted upon it, so $p_2(a) = p_1(a)$, $p_2(d) = p_1(d)$ and $u_2(a) = u_1(a)$. Equation (A7) then becomes,

$$\frac{W_2}{W_1} = \frac{\Re\left\{\left(p_1(a) + p_1(d)\right) u_1(a)^*\right\}}{\Re\{p_1(a) \, u_1(a)^*\}} \tag{A8}$$

Substituting equations (A3) and (A4) into (A8),

$$\frac{W_2}{W_1} = \frac{\Re\left\{Z_{r0}\left(1 + \frac{a}{d} e^{-jk(d-a)}\right)\right\}}{\Re\{Z_{r0}\}} = 1 + \frac{a}{d}\cos\left(k(d-a)\right) + \frac{1}{kd}\sin\left(k(d-a)\right) \tag{A9}$$

which, for d » a, can be approximated by,

$$\frac{W_2}{W_1} \approx 1 + \frac{\sin(kd)}{kd} \ , \ (d \gg a) \ . \tag{A9a}$$

Constant Pressure Sources

If the sources are assumed to operate as constant pressure sources, the surface velocity of the source changes with changes in external pressure exerted upon it so as to maintain constant surface pressure. In this case, the total pressure on the surface of one source when they are both operated can be equated to that when it is operated in isolation:

$$p_1(a) = p_2(a) + p_2(d) \ . \tag{A10}$$

Substituting equations (A3) and (A4),

$$Z_{r0} \, u_1(a) = Z_{r0}\left(1 + \frac{a}{d} e^{-jk(d-a)}\right) u_2(a) \ , \tag{A11}$$

and rearranging yields the surface velocity when both sources are operated in terms of that when one is operated,

$$u_2(a) = \frac{u_1(a)}{1 + \frac{a}{d} e^{-jk(d-a)}} \tag{A12}$$

Equation (A7) can now be written,

$$\frac{W_2}{W_1} = \frac{\Re\{p_1(a) \, u_2(a)^*\}}{\Re\{p_1(a) \, u_1(a)^*\}} \tag{A13}$$

Substituting equations (A3) and (A12),

$$\frac{W_2}{W_1} = \frac{\Re\left\{\dfrac{Z_{r0} \, |u_1(a)|^2}{\left(1 + \frac{a}{d} e^{-jk(d-a)}\right)^*}\right\}}{\Re\left\{Z_{r0} \, |u_1(a)|^2\right\}} = \frac{1 + \frac{a}{d}\cos\left(k(d-a)\right) + \frac{1}{kd}\sin\left(k(d-a)\right)}{1 + \left(\frac{a}{d}\right)^2 + \frac{2a}{d}\cos\left(k(d-a)\right)} \ , \tag{A14}$$

which, for $d \gg a$, can be approximated by.

$$\frac{W_2}{W_1} \approx 1 + \frac{\sin(kd)}{kd} \ , \ (d \gg a) \ . \tag{A14a}$$

which is identical to equation (A9a) for constant velocity sources

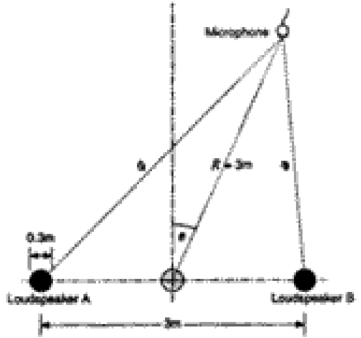

Figure 1 Geometry of Simulated Stereo Loudspeaker Pair

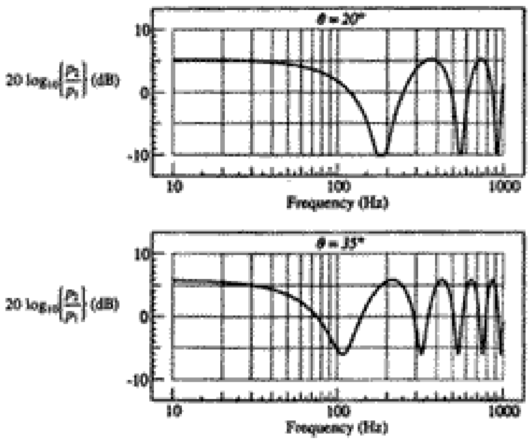

Figure 2 Simulation of Combined Off-Centre-Line Frequency Response of a Stereo Pair of Omnidirectional Velocity-Source Loudspeaker at Two Different Positions in an Anechoic Chamber Relative to that of a Single Loudspeaker Placed Mid-Way Between the Two – Loudspeaker Separation 3m – Loudspeaker Radius 0.15m – Microphone 3m from point mid-way between loudspeakers

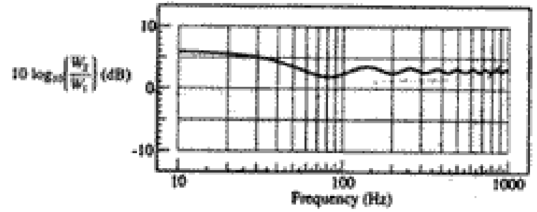

Figure 3 Simulation of the Combined Sound Power Output of the Loudspeaker Pair in Figure 2 Relative to that of a Single Loudspeaker

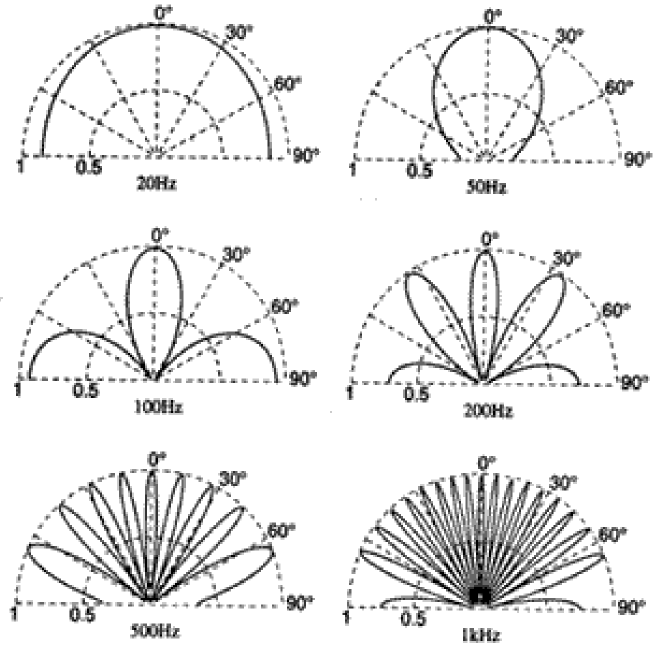

Figure 4 Simulation of Combined Far-Field Directivity of the Loudspeaker Pair in Figure 2

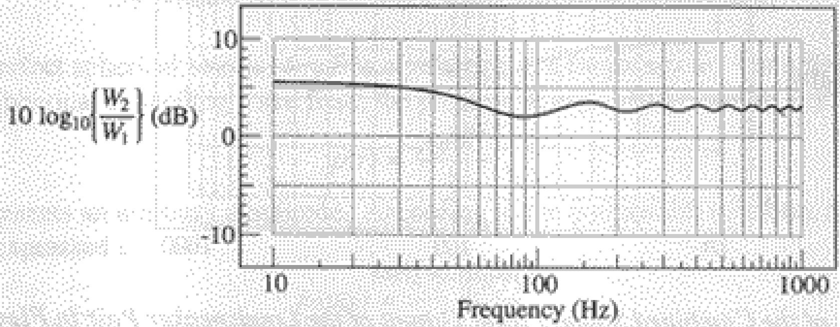

Figure 5 As Figure 3 but for a Pair of Pressure-Source Loudspeakers

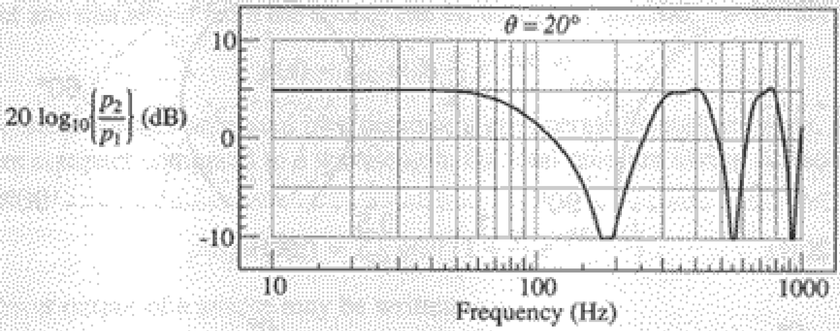

Figure 6 As Figure 2 but for a Pair of Pressure Source Loudspeakers

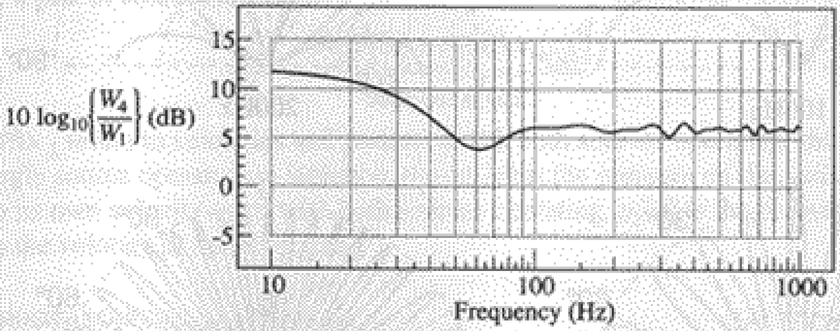

Figure 7 Simulation of Combined Sound Power Output of Four Omnidirectional Velocity-Source Loudspeakers Arranged in a Rectangle of 3m × 4m Relative to that of a Single Loudspeaker